

CONTENTS

PREFACE vii

1 INTRODUCTION 1

- 1-1 Definition of a Control System 1
- 1-2 Evolution of the Science of Control 3
- References 6

2 INTRODUCTORY EXAMPLE 9

- 2-1 Description of System 9
- 2-2 Construction of Mathematical Model 12
- 2-3 Linearization of Mathematical Model 16
- 2-4 Selection of Control Strategy 17
 - 2-4.1 Proportional Control 18
 - 2-4.2 Proportional-plus-derivative Control (PPD Control) 22
- 2-5 Summary 25
- Exercises 26

3 CONCEPTS OF STATE AND STATE VARIABLES 29

- 3-1 Introduction 29
- 3-2 Reduction of Differential Equations to Their Normal Form 31
- 3-3 Concepts of State and State Variables 33
- 3-4 State-model Description of Some Typical Engineering Systems 39
- 3-5 Definition of Linear Systems; Methods for Linearization 45
- 3-6 Effects of Disturbances on the State Differential Equation 53
- 3-7 Summary 53
- Exercises 56
- References 57

4 LINEAR ANALYSIS 59

- 4-1 Solution in Terms of the Matrix Exponential 60
- 4-2 Solution of Time-variant Linear Systems 67
- 4-3 Solution by Means of Laplace Transforms 69
- 4-4 Eigenvalues and Stability 71
- 4-5 Transfer Matrices and Transfer Functions 75
- 4-6 Derivation of State Models from Transfer Functions 77
- 4-7 Summary 83
- Exercises 83
- References 85

5 CONCEPTS OF CONTROLLABILITY AND OBSERVABILITY 87

- 5-1 Definitions 87
- 5-2 Invariance of the Eigenvalues to a Linear Transformation 93
- 5-3 Eigenvectors and Natural Modes 94
- 5-4 The Canonical Transformation 97
- 5-5 Criteria for Controllability 100
- 5-6 Observability and Observers 103
- 5-7 Effect of Multiple Eigenvalues 108
- 5-8 Summary 115
- Exercises 115
- References 117

6 LINEAR SERVOMECHANISMS—THE ANALYSIS PROBLEM 119

- 6-1 Definition 119
- 6-2 State-variable Description of Servomechanisms 122
- 6-3 Transfer-function Description of Linear Servos 129
- 6-4 Frequency-response Techniques 137
 - 6-4.1 Frequency-transfer Functions 139
 - 6-4.2 Bode Plots 144
 - 6-4.3 Nyquist Diagrams 146
 - 6-4.4 The Nyquist Stability Theorem 151
 - 6-4.5 Relative Stability; Gain and Phase Margins 155
- 6-5 Sensitivity and Error Analysis 158
 - 6-5.1 Errors due to Reference-input Changes 160
 - 6-5.2 Errors due to Parameter and Load Changes 163
 - 6-5.3 Concepts of Output Impedance and Compliance 170
- Exercises 172
- References 173

7 LINEAR SERVOMECHANISMS—THE SYNTHESIS PROBLEM 175

- 7-1 Performance Specifications 175
 - 7-1.1 Frequency-domain Specifications 177
 - 7-1.2 Time-domain Specifications 179
 - 7-1.3 Specification on Statistical Basis 183
- 7-2 Classification of Synthesis Methods 183
 - 7-2.1 The Trial-and-error Synthesis Technique 184
 - 7-2.2 Analytical Design Methods 186
 - 7-2.3 Optimum-control-system Design 187
- 7-3 Classical Design of Single-input-Single-output Servomechanisms 188
 - 7-3.1 The Analytical Approach—Two Examples 189
 - 7-3.2 The Root-locus Method (RLM) 196
 - Time Response from Root-locus Plot 197
 - Frequency Response from Root-locus Plot 199
 - Dominant Poles 200
 - Construction of Root Loci 204
 - 7-3.3 Synthesis by Trial and Error 213
 - 7-3.4 Cancellation and Algebraic Compensation 239
- 7-4 Synthesis of Multiple-input-Multiple-output Linear Servos 242
 - 7-4.1 Noninteraction 244
 - 7-4.2 Static Accuracy 245
 - 7-4.3 Stability 246
 - 7-4.4 Sensitivity Analysis 247

| | |
|-------------|-----|
| 7-5 Summary | 254 |
| Exercises | 255 |
| References | 260 |

8 NONLINEAR CONTROL SYSTEMS 263

| | |
|---|-----|
| 8-1 Introduction | 263 |
| 8-2 The State-space Analysis Method | 264 |
| 8-2.1 Phase Trajectories | 266 |
| 8-2.2 Singular Points or Equilibrium States | 267 |
| 8-2.3 Construction of Phase Trajectories—The Phase Plane | 269 |
| The Direct-solution Method | 270 |
| Phase-plane Construction by Means of the Isocline Method | 273 |
| 8-3 Stability of Nonlinear Systems | 278 |
| 8-3.1 Definitions | 278 |
| Local Stability | 279 |
| Finite and Global Stability | 286 |
| Stability Test by the Indirect Approach | 287 |
| Stability Test by Liapunov's Direct Method | 293 |
| 8-4 The Method of Harmonic Linearization (MLH) | 306 |
| 8-4.1 Harmonic Balance | 307 |
| 8-4.2 Describing Functions | 309 |
| Calculation of DF | 311 |
| Uses of DF | 317 |
| 8-4.3 Effects of High-frequency Bias Signal | 322 |
| Dual-input Describing Function (DIDF) | 322 |
| Spectral Composition of the Output | 325 |
| Computation of DIDF: The Concept of Equivalent Nonlinearity | 327 |
| 8-5 Summary | 332 |
| Exercises | 332 |
| References | 335 |

9 SIGNAL-MODULATED SYSTEMS 337

| | |
|--|-----|
| 9-1 Introduction | 337 |
| 9-2 Why Modulation? | 337 |
| 9-3 Theory of Amplitude-modulated Control Systems | 343 |
| 9-3.1 Spectral-energy Distribution | 343 |
| 9-3.2 AM-signal Compensation | 345 |
| Derivation of Transfer Function for Compensation Network | 345 |
| Design of AM Compensation Networks | 350 |
| 9-3.3 A Design Example | 353 |
| Description of System | 353 |
| The Sensors | 359 |
| The Torquer | 365 |
| The Signal Mixers (Resolvers) | 367 |
| The Complete System | 368 |
| 9-4 Sampled-data Systems | 371 |
| 9-4.1 The Sampling Process—Spectrum Analysis | 371 |
| 9-4.2 Demodulation of Sampled-data Signal | 374 |
| 9-4.3 Difference Equations and the Sampling Process | 376 |
| Stability | 379 |
| Response Type | 380 |
| 9-4.4 z-transform Theory | 384 |
| Definition of the Direct Transform | 384 |

| | |
|---|-----|
| Some Useful Properties of the z Transform | 387 |
|---|-----|

| | |
|-------------------------|-----|
| The Inverse z Transform | 391 |
|-------------------------|-----|

| | |
|-----------------------------------|-----|
| The Pulse Transfer Function (PTF) | 393 |
|-----------------------------------|-----|

| | |
|--|-----|
| Control-systems Analysis by Means of the z Transform | 405 |
|--|-----|

9-5 Summary 411

| | |
|-----------|-----|
| Exercises | 412 |
|-----------|-----|

| | |
|------------|-----|
| References | 413 |
|------------|-----|

10 OPTIMUM CONTROL—THE STATIC CASE 415

| | |
|--|-----|
| 10-1 Introduction | 415 |
| 10-2 Static-versus-dynamic Optimum Control | 417 |
| 10-3 Static Optimization | 423 |
| 10-3.1 The Mathematical-model Approach | 423 |
| Linear Performance Function with Linear Constraints | 427 |
| H Function Exhibiting Calculus Optima—The Method of Lagrange Multipliers | 429 |
| 10-3.2 Optimization by Experimentation | 434 |
| 10-3.3 Adaptive Controllers | 438 |
| Identification of the Plant Dynamics | 440 |
| Deciding on Proper Control Strategy | 440 |
| Modification of the Controller Parameters | 440 |
| Exercises | 441 |
| References | 442 |

11 OPTIMUM CONTROL—THE DYNAMIC CASE 445

| | |
|---|-----|
| 11-1 Introductory Remarks | 445 |
| 11-2 Optimization by Means of Calculus of Variations | 446 |
| 11-3 Dynamic Programming | 456 |
| 11-3.1 The Principle of Optimality (PO) | 459 |
| 11-4 Pontryagin's Maximum Principle (PMP) | 467 |
| 11-4.1 Derivation of Pontryagin's Maximum Principle (PMP) | 472 |
| x(t ₁) Completely Unconstrained, t ₁ Fixed | 473 |
| x(t ₁) Partly Constrained, t ₁ Fixed | 479 |
| The Time-optimal Case | 482 |
| Switching Curves | 487 |
| 11-5 Conclusions | 491 |
| Exercises | 492 |
| References | 494 |

12 THE ROLE OF THE COMPUTER IN DESIGN AND OPERATION OF CONTROL SYSTEMS 495

| | |
|---|-----|
| 12-1 Introductory Remarks | 495 |
| 12-2 The Analog Computer | 496 |
| 12-2.1 Computing Elements | 496 |
| 12-2.2 Manual-mode Control—Slow Operation | 496 |
| 12-2.3 Electronic-mode Control—Fast Operation | 498 |
| 12-2.4 The Concept of Simulation—The Analog as a Synthesis Tool | 499 |
| Simulation on the Basis of State Models | 499 |
| Simulation on the Basis of Transfer Functions | 504 |
| 12-2.5 Summary of Analog Features | 505 |
| 12-3 The Digital Computer | 506 |
| 12-3.1 Programming | 507 |

| | | |
|--------|--------------------------------|-----|
| 12-3.2 | Summary of Digital Features | 511 |
| 12-3.3 | Digital Simulation | 513 |
| 12-3.4 | Computer Control | 514 |
| 12-4 | Hybrid Computers | 517 |
| 12-4.1 | Reasons for Hybridization | 518 |
| 12-4.2 | Hybrid Computing Elements | 519 |
| 12-4.3 | Examples of Hybrid Computation | 522 |
| 12-5 | Summary | 530 |
| | Exercises | 530 |
| | References | 532 |

APPENDIXES

A ELEMENTS OF VECTOR AND MATRIX ALGEBRA 533

| | | |
|-------|------------------------------------|-----|
| A-1 | Vectors | 533 |
| A-1.1 | Special Vectors | 533 |
| A-1.2 | Elementary Vector Operations | 534 |
| A-1.3 | The Inner Vector Product | 535 |
| A-2 | Matrices | 537 |
| A-2.1 | Elementary Matrix Operations | 537 |
| A-2.2 | Special Matrices | 539 |
| A-2.3 | Determinants and Adjugate Matrices | 541 |
| A-2.4 | The Matrix Inverse | 542 |
| | References | 543 |

B FOURIER AND LAPLACE TRANSFORMS—A SUMMARY 545

| | | |
|-----|---|-----|
| B-1 | Fourier Series and Transforms—The Periodic Case | 545 |
| B-2 | Fourier Transforms—The Aperiodic Case | 548 |
| B-3 | Laplace Transforms | 550 |
| B-4 | Routh Criterion | 552 |
| | Reference | 553 |

INDEX 555