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PREFACE

This, the seventh edition of *Mathematical Methods for Physicists*, maintains the tradition set by the six previous editions and continues to have as its objective the presentation of all the mathematical methods that aspiring scientists and engineers are likely to encounter as students and beginning researchers. While the organization of this edition differs in some respects from that of its predecessors, the presentation style remains the same: Proofs are sketched for almost all the mathematical relations introduced in the book, and they are accompanied by examples that illustrate how the mathematics applies to real-world physics problems. Large numbers of exercises provide opportunities for the student to develop skill in the use of the mathematical concepts and also show a wide variety of contexts in which the mathematics is of practical use in physics.

As in the previous editions, the mathematical proofs are not what a mathematician would consider rigorous, but they nevertheless convey the essence of the ideas involved, and also provide some understanding of the conditions and limitations associated with the relationships under study. No attempt has been made to maximize generality or minimize the conditions necessary to establish the mathematical formulas, but in general the reader is warned of limitations that are likely to be relevant to use of the mathematics in physics contexts.

TO THE STUDENT

The mathematics presented in this book is of no use if it cannot be applied with some skill, and the development of that skill cannot be acquired passively, e.g., by simply reading the text and understanding what is written, or even by listening attentively to presentations by your instructor. Your passive understanding needs to be supplemented by experience in using the concepts, in deciding how to convert expressions into useful forms, and in developing strategies for solving problems. A considerable body of background knowledge