

Contents

Preface xvii
Acknowledgments xix
Acronyms..... xxi

Chapter 1 Introduction 1

1.1 Fundamentals of Composite Materials and Structures 2
1.2 Modeling of Composites 10
1.3 Description of the Book Layout..... 15
1.4 Suggestions on How to Use the Book 17

Chapter 2 Constituent Material Modeling..... 19

2.1 Reversible Models 25
2.1.1 Elasticity26
2.1.2 Ramberg-Osgood Nonlinear Elastic Constitutive Equations.....33
2.1.3 Viscoelasticity34
2.2 Irreversible Deformation Models 46
2.2.1 Incremental Plasticity47
2.2.2 Power-Law Creep49
2.2.3 Viscoplasticity50
2.3 Damage/Life Models 58
2.3.1 Continuum-Based Damage59
2.3.2 Interface Models80
2.4 Concluding Remarks 85

Chapter 3 Fundamentals of the Mechanics of Multiphase Materials..... 87

3.1 Introduction of Scales and Homogenization/Localization 88
3.2 Macromechanics versus Micromechanics 90
3.3 Representative Volume Elements (RVEs) and Repeating Unit Cells (RUCs) ... 94
3.4 Volume Averaging 95
3.5 Homogeneous Boundary Conditions..... 96
3.6 Average Strain Theorem..... 96

3.7 Average Stress Theorem.....	97
3.8 Determination of Effective Properties.....	98
3.8.1 Effective Elastic Moduli.....	98
3.8.2 Relations between Averages: A Direct Approach	99
3.8.3 Energy Approach.....	101
3.9 Mechanics of Composite Materials.....	103
3.9.1 The Voigt Approximation.....	104
3.9.2 The Reuss Approximation	105
3.9.3 Combined Voigt and Reuss Approximations	105
3.9.4 Hill's Theorem	105
3.9.5 Bounding Theorems.....	108
3.9.6 The Dilute Approximation.....	109
3.9.7 The Composite Spheres Assemblage Model.....	111
3.9.8 The Self-Consistent Scheme (SCS).....	113
3.9.9 The Generalized Self-Consistent Scheme (GSCS)	114
3.9.10 The Differential Scheme (DS).....	116
3.9.11 The Mori-Tanaka (MT) Theory.....	118
3.9.12 Eshelby Equivalent Inclusion Method.....	121
3.9.13 The Strain Concentration Tensors for the Self-Consistent (SCS) and Differential Schemes (DS).....	125
3.9.14 The Strain Concentration Tensor for the Mori-Tanaka (MT) Method	126
3.9.15 Green's Function/Fourier Series.....	129
3.9.16 Numerical Methods.....	130
3.10 Comparison of Various Micromechanics Methods for Continuous Reinforcement.....	130
3.11 Levin's Theorem: Extraction of Effective CTE from Mechanical Effective Properties	134
3.11.1 Multiphase Composite	134
3.11.2 Two-Phase Composite	136
3.11.3 The Generalization of Levin's Theorem: Transformation Fields	138
3.12 The Self-Consistent Scheme (SCS) and Mori-Tanaka (MT) Method for Inelastic Composites	144
3.13 Concluding Remarks	145
Chapter 4 The Method of Cells Micromechanics	147
4.1 The MOC for Continuously Fiber-Reinforced Materials (Doubly Periodic)...	148
4.1.1 Thermomechanical Formulation	149
4.1.2 Thermal Conductivities	176

4.1.3 Specific Heats	178
4.1.4 MOC with Imperfect Bonding	180
4.2 The Method of Cells for Discontinuously Fiber-Reinforced Composites (Triply Periodic)	181
4.2.1 Thermomechanical Formulation	181
4.2.2 Thermal Conductivity	187
4.3 Applications: Unidirectional Continuously Reinforced Composites.....	189
4.3.1 Effective Elastic Properties	189
4.3.2 Coefficients of Thermal Expansion.....	195
4.3.3 Specific Heat.....	197
4.3.4 Yield Surfaces of Metal Matrix Composites	198
4.3.5 Inelastic Response of Metal Matrix Composites	205
4.4 Applications: Discontinuously Reinforced (Short-Fiber) Composites	212
4.4.1 Effective Elastic Properties	212
4.5 Applications: Randomly Reinforced Materials.....	217
4.6 Concluding Remarks	224
Chapter 5 The Generalized Method of Cells Micromechanics	227
5.1 GMC for Discontinuous Reinforced Composites (Triple Periodicity).....	229
5.1.1 Thermomechanical Formulation	229
5.1.2 Thermal Conductivities	243
5.1.3 Electric Conductivity	243
5.1.4 Specific Heat.....	244
5.1.5 Computationally Efficient Reformulation.....	246
5.2 Specialization of GMC to Continuously Reinforced Composites (Double Periodicity)	259
5.3 Applications	267
5.3.1 Effective Properties.....	267
5.3.2 Local Fields	271
5.3.3 Response of Continuously Reinforced Unidirectional Composites	277
5.3.4 Discontinuous Reinforced Composites	323
5.3.5 Modeling Woven PMC Composites.....	336
5.4 Concluding Remarks	349
Chapter 6 The High-Fidelity Generalized Method of Cells Micromechanics	351
6.1 Three-Dimensional (Triply Periodic) High-Fidelity Generalized Method of Cells with Imperfect Bonding Between the Phases	353
6.1.1 Governing Equations	355
6.1.2 Displacement Expansion	358

6.1.3 Constitutive Equations.....	359
6.1.4 Determination of the Unknown Microvariables	360
6.1.5 Imperfect Bonding.....	363
6.1.6 Summary of Global Equations	364
6.2 Specialization to Double Periodicity (for Continuous Fibers, Anisotropic Constituents, and Imperfect Bonding)	368
6.2.1 Governing Equations	368
6.2.2 Displacement Expansion	371
6.2.3 Constitutive Equations.....	372
6.2.4 Determination of the Unknown Microvariables	373
6.2.5 Imperfect Bonding.....	374
6.2.6 Summary of Equations	375
6.3 Reformulation of the Two-Dimensional (Doubly Periodic) HFGMC with Debonding and Inelasticity Effects	376
6.3.1 Governing Equations	377
6.3.2 Displacement Expansion	377
6.3.3 Constitutive Equations.....	378
6.3.4 The Determination of the Displacement's Microvariables in Terms of the Average Surface Displacements	382
6.3.5 The Determination of the Average Surface Traction in Terms of the Average Surface Displacements	385
6.3.6 Perfect Bonding: Reduction of the Number of Unknowns	389
6.3.7 Imperfect Bonding.....	389
6.3.8 Computational Impact of Reformulation	392
6.4 Contrast Between HFGMC and Finite Element Analysis (FEA).....	395
6.5 Isoparametric Subcell Generalization	396
6.6 Doubly Periodic HFGMC Applications.....	403
6.6.1 Effective Elastic Properties and Coefficient of Thermal Expansion (CTE)	403
6.6.2 Local Fields: HFGMC versus Eshelby Solution.....	406
6.6.3 Local Fields: HFGMC versus FEA.....	409
6.6.4 Graphite/Aluminum Composite Global Response: HFGMC versus MCCM	411
6.6.5 Titanium Composite (SiC/TIMETAL 21S) Response: Discretization Convergence Study	412
6.6.6 Fiber-Matrix Debonding.....	415
6.6.7 Woven Composite Example: 5HS SiC/SiC	419
6.6.8 Progressive Damage/Failure of Polymeric Composite	423
6.7 Triply Periodic Applications	431

6.7.1 Lattice Block/Open Cell.....	431
6.7.2 Micromechanical Analysis of Foams with Internal Pore Pressure.....	437
6.8 Concluding Remarks	444
Chapter 7 Multiscale Modeling of Composites	447
7.1 Introduction.....	447
7.2 Multiscale Analysis Using Lamination Theory	459
7.2.1 Transverse Tensile and Creep Response of an SiC/Ti Composite	465
7.2.2 Fatigue of SiC/Ti-15-3.....	472
7.2.3 High-Temperature Creep of a Woven Ceramic Matrix Composite.....	477
7.2.4 Progressive Failure of PMC Laminates	481
7.3 HyperMAC.....	488
7.3.1 Progressive Failure of a T-Stiffened Composite Panel	489
7.3.2 Fatigue Life Prediction of a Foam Core Sandwich Beam.....	491
7.4 Multiscale Generalized Method of Cells (MSGMC).....	498
7.4.1 Micro Scale (Constitutive Modeling).....	501
7.4.2 Meso Scale (Multiphase Material)	501
7.4.3 Macro Scale (Weave)	503
7.4.4 Sensitivity Study of PMC Parameters across Scales	505
7.4.5 Triaxially Braided PMC	518
7.5 FEAMAC	522
7.5.1 Stochastic Fiber Breakage of a Longitudinally Reinforced SiC/Ti Metal Matrix Composite (MMC).....	523
7.5.2 Progressive Failure of a Notched Composite Laminate Plate.....	533
7.5.3 Delamination Modeling.....	536
7.6 Concluding Remarks	539
Chapter 8 Fully Coupled Thermomicromechanical Analysis of Multiphase Composites	541
8.1 Introduction.....	541
8.2 Classical Thermomicromechanical Analysis	542
8.2.1 The Homogenization Procedure.....	542
8.2.2 Solution of the Repeating Unit Cell Problem.....	547
8.3 Fully Coupled Thermomicromechanical Analysis	550
8.3.1 The Energy Equation.....	551
8.3.2 The Homogenized Energy Equation	552
8.3.3 The Fully Coupled Thermomechanical Solution of the Repeating Unit Cell Problem.....	557
8.4 Applications	560

8.4.1 Monolithic Aluminum Subjected to Cyclic Loading	565
8.4.2 $\text{Al}_2\text{O}_3/\text{Al}$ Subjected to Axial Cyclic Loading	566
8.4.3 $\text{Al}_2\text{O}_3/\text{Al}$ Subjected to Transverse Cyclic Loading	568
8.4.4 $\text{Al}_2\text{O}_3/\text{Al}$ Subjected to Axial and Transverse Shear Cyclic Loading	571
8.4.5 Full Thermomechanical Coupling with Damage	574
8.5 Concluding Remarks	576
Chapter 9 Finite Strain Micromechanical Modeling of Multiphase Composites	577
9.1 Introduction	578
9.2 Finite Strain Generalized Method of Cells (FSGMC)	581
9.2.1 Material Representation of the Monolithic Thermoelastic Material	581
9.2.2 FSGMC Formulation for Thermoelastic Composites	585
9.3 Applications Utilizing FSGMC	592
9.3.1 Porous Material Undergoing Large Deformation Subjected to Hydrostatic Loading	592
9.3.2 Class I—Harmonic Material	594
9.3.3 Class II	595
9.3.4 Class III—Generalized Varga Material	596
9.3.5 Discontinuous Reinforced Composites	597
9.3.6 Continuous Reinforced Composites	598
9.4 Finite Strain High-Fidelity Generalized Method of Cells (FSHFGMC) for Thermoelastic Composites	599
9.4.1 The Homogenization Procedure	600
9.4.2 Method of Solution of the RUC Problem	603
9.5 Applications Utilizing FSHFGMC	608
9.5.1 Hyperelastic Matrix Composites Exhibiting the Mullins Damage Effect	608
9.5.2 Thermoelastic Composites	617
9.5.3 Viscoelastic Composites	620
9.5.4 Thermoviscoelastic Composites	639
9.5.5 Thermoelastoplastic Composites	653
9.5.6 Elastoplastic Composites with Evolving Damage	659
9.5.7 Thermoviscoplastic Composites	669
9.5.8 Optimization of Porous Microstructures	675
9.6 Concluding Remarks	676
Chapter 10 Micromechanical Analysis of Smart Composite Materials	677
10.1 Introduction	678
10.2 Electro-Magneto-Thermo-Elastic Composites	680

10.2.1 Effective Behavior of Unidirectional Electro-Magneto- Thermo-Elastic Composites via GMC	680
10.2.2 Effective Behavior of Unidirectional Electro-Magneto- Thermo-Elastic Composites via the High-Fidelity Generalized Method of Cells (HFGMC)	695
10.2.3 Thermo-Electro-Magneto-Elasto-Plastic Lamination Theory	699
10.3 Hysteresis Behavior of Ferroelectric Fiber Composites	706
10.3.1 The Modeling of the Monolithic Ferroelectric Material	707
10.3.2 Incremental Micromechanics Analysis	712
10.4 The Response of Electrostrictive Composites	714
10.4.1 The Modeling of the Monolithic Electrostrictive Material	715
10.4.2 Incremental Micromechanical Analysis	717
10.5 Analysis of Magnetostrictive Composites	718
10.6 Nonlinear Electro-Magneto-Thermo-Elastic Composites	718
10.7 Shape Memory Alloy Fiber Composites	720
10.7.1 Constitutive Models for SMA Response	721
10.8 Shape Memory Alloy Fiber Composites Undergoing Large Deformations ..	731
10.8.1 Finite-Strain Shape Memory Alloy Constitutive Equations	731
10.8.2 Finite-Strain Constitutive Equations for SMA Fiber Composites	736
10.9 Applications	737
10.9.1 Piezoelectric Effective Constants	737
10.9.2 Electromagnetic Effective Constants	739
10.9.3 Multiscale Analysis of a Hybrid Smart/Metal Matrix Composite Laminate	740
10.9.4 Macroscopic Hysteresis Response of Ferroelectric Fiber Composites	748
10.9.5 Porous Lead Magnesium Niobate (PMN) Electrostrictive Material Behavior	751
10.9.6 Nonlinear Electro-Magneto-Thermo-Elastic $\text{LiNbO}_3/\text{PVDF}$ Composite Response	754
10.9.7 Unidirectional SMA Fiber Composite Response Using Lagoudas (and Coworkers) Model	755
10.9.8 Response of SMA/Epoxy Unidirectional Composite Using Auricchio (and Coworkers) Model	759
10.9.9 Response of SMA/Epoxy Unidirectional Composite Using the Two-Way Shape Memory Effect Model	760
10.9.10 Response of SMA/Al Unidirectional Composite Undergoing Large Deformations	763
10.9.11 Dynamic Response of SMA Composite Plates	765

10.9.12 Thermal Buckling of Activated SMA Composite Plates	767
10.10 Concluding Remarks	770
Chapter 11 Higher-Order Theory for Functionally Graded Materials	773
11.1 Background and Motivation	775
11.2 Generalized Three-Directional HOTFGM	779
11.2.1 Model Overview	779
11.2.2 Thermal Analysis.....	781
11.2.3 Mechanical Analysis	786
11.2.4 Higher-Order Theory versus Finite Element Analysis	793
11.3 Specialization of the Higher-Order Theory	794
11.3.1 Two-Directional HOTFGM	795
11.3.2 One-Directional HOTFGM	798
11.4 Higher-Order Theory for Cylindrical Functionally Graded Materials (HOTCFGM)	801
11.4.1 Thermal Analysis.....	802
11.4.2 Mechanical Analysis	804
11.5 HOTFGM Applications	805
11.5.1 Finite Element Validation.....	806
11.5.2 Microstructural Effects	814
11.5.3 Microstructural Tailoring.....	833
11.5.4 Microstructural Optimization	840
11.5.5 Internally Cooled Plate.....	848
11.5.6 Smart Materials in HOTFGM	855
11.6 HOTCFGM Applications	863
11.6.1 Case I: Thin-Walled Cylinder Subjected to Internal Pressure	864
11.6.2 Case II: Thin-Walled Cylinder Subjected to Temperature Gradient...	865
11.6.3 Case III: Transient Thermal Loading.....	865
11.6.4 Thrust Cell Liner Response.....	867
11.7 Concluding Remarks	876
Chapter 12 Wave Propagation in Multiphase and Porous Materials	879
12.1 Full Three-Dimensional Theory	880
12.2 Specialization to Two-Dimensional Theory for Thermoelastic Materials	893
12.3 The Inclusion of Inelastic Effects	902
12.4 Two-Dimensional Wave Propagation with Full Thermoelastic Coupling.....	905
12.5 Applications	910
12.5.1 Acoustic Evaluation of Foam Sandwich Panels	910
12.5.2 Acoustic Evaluation of Heterogeneous Panels	913

12.5.3 Temperature Field Induced by Dynamic Stresses on a Cracked Composite	916
12.6 Concluding Remarks	928
Chapter 13 Micromechanics Software	929
13.1 Accessing the Software	930
13.2 Method of Cells Source Code	930
13.2.1 Method of Cells Source Code Example Problem.....	932
13.3 MAC/GMC 4.0	932
13.3.1 Getting Started.....	933
13.3.2 Executing MAC/GMC 4.0 Problems	933
13.3.3 Format of the MAC/GMC 4.0 Input File.....	935
13.3.4 MAC/GMC 4.0 Example Problem.....	936
13.3.5 MAC/GMC 4.0 Example Problem Input	937
13.4 Concluding Remarks	941
References	943
Index	973