
CONTENTS

Preface	xiii
1 Introduction	1
1.1 Background and Motivation / 3	
1.2 Objectives / 4	
1.3 Organization of Book / 4	
1.4 Notation Conventions / 5	
References / 9	
2 Basic Equations of Continuum Mechanics	11
2.1 Displacement and Deformation / 11	
2.2 Stresses and Equilibrium / 15	
2.3 Energy, Work, and Thermodynamic Potentials / 18	
2.4 Constitutive Laws / 22	
2.5 Boundary Value Problems for Small-Strain Linear Elasticity / 34	
2.6 Integral Representations of Elasticity Solutions / 35	
Problems / 43	
Appendix 2.A / 44	
Appendix 2.B / 44	
Appendix 2.C / 46	
References / 47	
Suggested Readings / 48	
3 Eigenstrains	49
3.1 Definition of Eigenstrains / 49	
3.2 Some Examples of Eigenstrains / 52	
3.3 General Solutions of Eigenstrain Problems / 53	

- 3.4 Examples / 56
- Problems / 66
- Appendix 3.A / 67
- Appendix 3.B / 70
- References / 70
- Suggested Readings / 70

4 Inclusions and Inhomogeneities

71

- 4.1 Definitions of Inclusions and Inhomogeneities / 71
- 4.2 Interface Conditions / 73
- 4.3 Ellipsoidal Inclusion with Uniform Eigenstrains
(Eshelby Solution) / 77
- 4.4 Ellipsoidal Inhomogeneities / 84
- 4.5 Inhomogeneous Inhomogeneities / 88
- Problems / 91
- Appendix 4.A / 92
- Appendix 4.B / 93
- Suggested Readings / 98

5 Definitions of Effective Moduli of Heterogeneous Materials

99

- 5.1 Heterogeneity and Length Scales / 99
- 5.2 Representative Volume Element / 102
- 5.3 Random Media / 102
- 5.4 Macroscopic Averages / 108
- 5.5 Hill's Lemma / 111
- 5.6 Definitions of Effective Modulus of Heterogeneous
Media / 113
- 5.7 Concentration Tensors and Effective Properties / 116
- Problems / 119
- Suggested Readings / 119

6 Bounds for Effective Moduli

120

- 6.1 Classical Variational Theorems in Linear
Elasticity / 120
- 6.2 Voigt Upper Bound and Reuss Lower Bound / 123

- 6.3 Extensions of Classical Variational Principles / 126
- 6.4 Hashin–Shtrikman Bounds / 136
 - Problems / 145
 - Appendix 6.A / 147
 - References / 152
 - Suggested Readings / 153

7 Determination of Effective Moduli 154

- 7.1 Basic Ideas of Micromechanics for Effective Properties / 154
- 7.2 Eshelby Method / 158
- 7.3 Mori–Tanaka Method / 163
- 7.4 Self-Consistent Methods for Composite Materials / 169
- 7.5 Self-Consistent Methods for Polycrystalline Materials / 173
- 7.6 Differential Schemes / 179
- 7.7 Comparison of Different Methods / 183
 - Problems / 193
 - Suggested Readings / 195

8 Determination of the Effective Moduli—Multiinclusion Approaches 196

- 8.1 Composite-Sphere Model / 196
- 8.2 Three-Phase Model / 200
- 8.3 Four-Phase Model / 203
- 8.4 Multicoated Inclusion Problem / 204
 - Problems / 206
 - Appendix 8.A / 207
 - Appendix 8.B / 208
 - Appendix 8.C / 210
 - References / 211
 - Suggested Readings / 212

9 Effective Properties of Fiber-Reinforced Composite Laminates 214

- 9.1 Unidirectional Fiber-Reinforced Composites / 214

9.2	Effective Properties of Multilayer Composites /	226
9.3	Effective Properties of a Lamina /	234
9.4	Effective Properties of a Laminated Composite Plate /	238
	Problems /	240
	Appendix 9.A /	240
	References /	243
	Suggested Readings /	244
10	Brittle Damage and Failure of Engineering Composites	245
10.1	Imperfect Interfaces /	245
10.2	Fiber Bridging /	259
10.3	Transverse Matrix Cracks /	266
	Problems /	277
	Appendix 10.A /	277
	References /	278
	Suggested Readings /	279
11	Mean Field Theory for Nonlinear Behavior	280
11.1	Eshelby's Solution and Kröner's Model /	283
11.2	Applications /	299
11.3	Time-Dependent Behavior of Polycrystalline Materials: Secant Approach /	312
	Problems /	316
	References /	316
12	Nonlinear Properties of Composites Materials: Thermodynamic Approaches	319
12.1	Nonlinear Behavior of Constituents /	321
12.2	Effective Potentials /	328
12.3	The Secant Approach /	331
	Problems /	342
	Suggested Readings /	345

13	Micromechanics of Martensitic Transformation in Solids	347
13.1	Phase Transformation Mechanisms at Different Scales / 350	
13.2	Application: Thermodynamic Forces and Constitutive Equations for Single Crystals / 367	
13.3	Overall Behavior of Polycrystalline Materials with Phase Transformation / 373	
	Problems / 377	
	References / 379	
	Suggested Readings / 379	
	Index / 381	