

Stress Analysis of Fiber-Reinforced Composite Materials

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Preface to the Updated Edition

This updated edition of *Stress Analysis of Fiber Reinforced Composite Materials* contains the same material as the original, with important exceptions. Typographical errors, identified by readers and reviewers, have been corrected. Changes were made to a number of figures to increase their clarity. Equations were modified to increase consistency throughout the text, and wording in many of the Exercises was edited to clarify what is being asked.

The book is intended as an introductory text for upper level undergraduate engineering students or first year graduate students. However, the book has proven to be useful for practicing engineers who find it necessary to understand the behavior of composite materials. The book emphasizes the mechanics of a stress and deformation analysis of fiber-reinforced materials as opposed to, for example, a materials science viewpoint. Exercises, including computer oriented exercises, are included within most chapters.

One key feature of the book, and one that sets it apart from other books on the subject, remains the series of example problems that are discussed throughout the text, starting with rather simple problems in Ch.4 that are then expanded upon in subsequent chapters. This series of problems uses the same material properties throughout so the impact of the elastic and thermal expansion properties for a single layer of fiber-reinforced material, in this case graphite-epoxy, on the stress, strains, elastic properties, thermal expansion, and failure stresses of cross-ply and angle-ply symmetric and unsymmetric laminates can be evaluated. Furthermore, calculations for various steps of the stress and deformation analysis of a laminate discussed in later chapters require combining simpler calculations from previous chapters, so using the same material properties throughout allows the example problems to conveniently build upon each other as the chapters progress and illustrate how concepts are linked. To provide examples of the unique and sometimes complicated deformation properties of fiber-reinforced materials, examples are sometimes reworked using the properties of aluminum, and the stresses and deformations of aluminum compared with those of graphite-epoxy. Users of the text have commented on how much they like the number and style of the example problems.